International Journal of Theoretical Physics, Vol. 39, No. 5, 2000

Calogero Conjecture and Hubble Law

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Received October 15, 1999

Acceptance of the Calogero hypothesis on the "cosmic origin of quantization" in the framework of Nelson stochastic mechanics would imply that the age of universe is larger than the standard quantum mechanical interpretation of the redshift measurement implies. This is due to variation of h with the radius of the universe and thus with time.

1. INTRODUCTION

According to stochastic mechanics [1–6] (see in particular ref. 3), the quantum behavior of nature is due to the interaction of a universal "background noise" with any existing particle or body.

Stochastic mechanics is mathematically equivalent to quantum mechanics, but the background noise hypothesis calls for an attempt to go beyond standard quantum mechanics and investigate if such a noise is something more than a convenient mathematical artifact, i.e., if it has a physical meaning and even more if one can understand it as due to some kind of fundamental physical interaction.

As the background noise would interact with absolutely any form of matter (or energy), it would be natural to think that it should be related to the gravitational interaction (a different mechanism, considering interaction with the cosmic background radiation, is considered in ref. 7); however, it is immediately clear that for dimensional reasons it is impossible to obtain the Planck constant h in terms of the Newton gravitational constant G and the speed of light c. Even introducing some fundamental unit of mass—e.g., the total mass of the universe M, or at the other extreme the mass m of the hydrogen atom or of the nucleon—does not help: dimensionally one could

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0020-7748/00/0500-1339$18.00/0

obtain h in this way, but to get the right numerical value one needs a dimensionless constant which is far from being of order one; thus one needs another constant to introduce a further time or space scale (see below for the successful Calogero choice of such an additional constant).

If one believes that the gravitational interaction at the origin of the background noise and thus of quantum behavior is the one with the whole universe (i.e., the fact that no physical system is really isolated) and thus mainly with distant masses, it is natural to think that the appropriate constant to introduce is either the age of universe T or its radius R, or the Hubble constant H, or finally (assuming this is nonzero) the cosmological constant Λ.

It was observed by Calogero [8] that if one tries to substantiate these qualitative consideration by semiquantitative ones, it is indeed possible to "predict" the value of Planck constant as

$$h \approx \alpha G^{1/2} m^{3/2} R^{1/2} \tag{1}$$

[this is Eq. (3.4b) of ref. 8] with α a numerical constant of order one. Here G is the Newton gravitation constant, m the mass of the hydrogen atom, and R the radius of the universe (obviously the latter is much less precisely known); with the values

$$m = 1.67 \times 10^{-24}\ \text{g}, \quad G = 6.67 \times 10^{-8}\ \text{cm}^3\ \text{g}^{-1}\ \text{s}^{-2}, \quad R = 10^{28}\ \text{cm} \tag{2}$$

Calogero obtains for $\alpha = 1$ the remarkable estimate

$$h \simeq 6 \times 10^{-26}\ \text{cm}^2\ \text{gs}^{-1} \tag{3}$$

If we assume that (1) is verified, it follows that h changes with time, and that at time t its value is

$$h = A[R(t)]^{1/2} \tag{4}$$

where $R(t)$ is the radius of the universe at time t, and $A \approx 6 \times 10^{-40}$ cm$^{3/2}$ gs^{-1} is a constant which can be estimated from the present value of h, which we will from now on denote as h_0.

Remark 1. We recall that in the cosmological literature one often uses, instead of $R(t)$, the quantity $a(t) = R(t)/R_0$, where R_0 is the present radius of the universe; $a(t)$ is the scale factor for R and its present value is by definition $a(t_0) = 1$. With this, (4) reads $h = \tilde{A}\sqrt{a(t)}$, where $\tilde{A} = A\sqrt{R_0}$ is by definition equal to h_0; thus, $h(t) = h_0\sqrt{a(t)}$.

In this note we discuss the cosmological consequences of the Calogero conjecture, and in particular its impact on the Hubble law [9]; see Remarks 5 and 6 about other questions related to cosmological constraints on the Calogero conjecture.

The reason to expect a modification of the Hubble law, and for checking if this can be compatible with observations, is the following: if we assume (4), then radiation from distant objects would have been emitted with a different value of h and thus with a frequency different from the one usually assumed; in particular, the dominant radiation corresponding to the first spectral line of hydrogen would have been emitted with a frequency $B/h^3(t)$ [see (15)], which is higher than the present one. Therefore, when we compare the observed frequency of radiation from distant objects, the redshift would be higher than in the standard interpretation. This effect increases more than linearly for greater distances: we expect the linear relation $v = Hr$ should be modified into $v = H_0 r + H_1 r^2 + \cdots$.

Thus, the Calogero conjecture would imply that the existing observational data for the relation between redshift and distance, even for relatively small distances, should be reinterpreted as showing an accelerating expansion of the universe. Notice that this implies, in particular, that the age of the universe would be greater than estimated on the basis of the standard interpretation of cosmological data.

We stress that this modification of the Hubble law could be inferred in particular from data for objects at relatively small distances, i.e., in a region where the direct measurement of distance is possible; this is therefore not subject to experimental uncertainties of the same nature as those necessarily entering the discussion of data referring to extremely far regions of the universe—for which distance measurements necessarily rely on a chain of physical theories—like those which were recently considered in various works [10–12] and also led to postulating accelerating expansion.

In the following we will try to transform these qualitative considerations into quantitative ones. We will limit ourselves to a homogeneous and isotropic matter-dominated universe subject to a uniform expansion. This is in contradiction with the expected (and actual) result of an accelerating expansion, but will provide a suitable arena for exact computations and discussion of correction terms.

2. DISTANCE AND REDSHIFT I

According to relativity, an electromagnetic wave emitted with frequency ν_{em} by a body traveling directly away from us with speed v is observed as having a frequency ν_{ob}, where

$$\nu_{\mathrm{ob}} = \nu_{\mathrm{em}} \frac{1 - v/c}{1 - v^2/c^2} \tag{5}$$

This leads to a measurement of the recession speed in terms of the emitted and observed frequencies of the wave:

$$\frac{v}{c} = \frac{\nu_{\mathrm{em}}^2 - \nu_{\mathrm{ob}}^2}{\nu_{\mathrm{em}}^2 + \nu_{\mathrm{ob}}^2} \tag{6}$$

For later discussion we will express both ν_{em} and ν_{ob} in terms of a reference frequency ν_0, and v in terms of c, as

$$\nu_{\mathrm{em}} = \mu\nu_0, \qquad \nu_{\mathrm{ob}} = \beta\nu_0, \qquad v = wc \tag{7}$$

In this way (6) reads

$$w \equiv \frac{v}{c} = \frac{\mu^2 - \beta^2}{\mu^2 + \beta^2} \tag{8}$$

This is indeed the way in which, measuring the redshift, we estimate the speed of distant stars or galaxies: if we measure the observed frequency of the spectral line corresponding e.g., to the transition between the two first levels of hydrogen and compare it with the emission frequency ν_0 (here $\mu = 1$), we have

$$w_{\mathrm{q}} = \frac{1 - \beta^2}{1 + \beta^2} \tag{9}$$

We will refer to this as the “standard quantum mechanical” estimation of recession speed.

Remark 2. In the astrophysical literature, the redshift is described by the quantity $Z := (\lambda_{\mathrm{ob}} - \lambda_{\mathrm{em}})/\lambda_{\mathrm{em}}$, where λ is the wavelength of the electromagnetic wave, $\lambda = c/\nu$. In terms of the frequency ν of the wave, one can write this as $Z = [(\nu_{\mathrm{em}} - \nu_{\mathrm{ob}})/\nu_{\mathrm{ob}}]$.

It is well known that there is an approximate linear relation between ν_{q} and the distance r of the recessing objects, expressed as

$$v_{\mathrm{q}} = Hr \tag{10}$$

where H is the Hubble constant [it should be stressed that H is constant in space, but not in time; thus we should more precisely write $H(t)$]; its present value is not unanimously agreed on and different groups give different estimates of it; according to ref. 9 its value is

$$H \approx 55\ (\mathrm{km/s})/\mathrm{Mps} \approx 1.9 \times 10^{-21}\ \mathrm{s}^{-1} \tag{11$'$}$$

while both ref. 13 and more recent measurements [10–12] propose higher values, not far from

$$H \approx 70\ (\mathrm{km/s})/\mathrm{Mps} \approx 2.4 \times 10^{-21}\ \mathrm{s}^{-1} \tag{11$''$}$$

We introduce the constant $\gamma := c/H$; using (11$''$) for H, we get $\gamma \simeq 1.25 \times 10^{31}$ cm. From (6) and (10) we have then

$$r = \frac{v}{H} = \gamma\left(\frac{\mu^2 - \beta^2}{\mu^2 + \beta^2}\right) \tag{12}$$

Using the estimate (9) for ν_q, corresponding to $\mu = 1$, we get

$$r_q = \gamma\left(\frac{1 - \beta^2}{1 + \beta^2}\right) \tag{13}$$

Remark 3. We stress that for objects at a distance r such that it can be measured directly (e.g., via parallax), (13) is an experimentally verified relation between the observable quantities r and β.

Remark 4. It should be noted that the validity of (10)—which expresses homogeneous expansion of the universe—at high redshift (very far objects) has been questioned by recent measurements on far supernovae, opening problems concerning the age of the universe and the existence of a nonzero cosmological constant (see, however, refs. 10–12). We will assume validity of (10), thus implicitly restricting consideration to regions of the universe which are not too far away. Actually the Remark 3 provides another (at least equally good) reason for such a restriction.

3. DISTANCE AND RED SHIFT II

If now we adopt Calogero's conjecture, we have to consider that the photons emitted by the distant stars or galaxies had been emitted with a different value of h, i.e., that $\nu_{em} \neq \nu_0$, $\mu = (\nu_{em}/\nu_0) \neq 1$.

Focusing on the transition corresponding to the first two levels of the hydrogen atom, we know from quantum mechanics that this corresponds to an energy difference

$$\varepsilon_0 = \frac{3}{4}\frac{m_e\, e^4\, \pi^2}{2}\frac{1}{h^2} \equiv \frac{B}{h^2} \tag{14}$$

(where e, m_e are the charge and mass of the electron) and the frequency of the emitted electromagnetic wave is given by $h\nu_{em} = \varepsilon_0$; therefore the emitted frequency is

$$\nu_{em} = \frac{\varepsilon_0}{h} = \frac{B}{h^3} \tag{15}$$

According to (4), i.e., to the Calogero conjecture, we have

$$\nu_{em} = \frac{B}{A^3 R_{em}^{3/2}} = \nu_0\left(\frac{R_0}{R_{em}}\right)^{3/2} = \nu_0\,\frac{1}{[a(t_{em})]^{3/2}} \tag{16}$$

where $R_0 = R(t_0)$ is the present radius of the universe, $R_{em} = R(t_{em})$ is the radius of the universe at the time the radiation was emitted, and ν_0 is the frequency corresponding to this transition among hydrogen atom levels with the present value of h, i.e., $\nu_0 = B/h_0^3$. It follows from (16) that

$$\mu^2 := \left(\frac{\nu_{em}}{\nu_0}\right)^2 = \left(\frac{R_0}{R_{em}}\right)^3 \tag{17}$$

This value should be used in (8) to obtain an estimation of the recession speed according to the Calogero conjecture:

$$w_{cal} = \frac{1 - (R_{em}/R_0)^3\beta^2}{1 - (R_{em}/R_0)^3\beta^2} \tag{18}$$

The value of the ratio $R_{em}/R_0 \equiv R(t_0 - \tau)/R(t_0) = \mu^{-2/3}$ depends on our model for the expansion of the universe. Notice that, as already implicitly mentioned in (16), this ratio is just $a(t_{em})$, with $a(t)$ the scale factor defined earlier.

In order to use (18), we need, of course, to know R_{em}; if we know the distance r of the object which emitted the electromagnetic radiation, we know that the wave has traveled for a time τ. Notice that in an expanding universe this is *not* simply $\tau = r/c$: this can be seen as due to the variation of the metric with time through the scale factor $a(t)$, and the derivation of $\tau(r)$ is briefly discussed below.

The equation determining τ is simply

$$\int_{t_0-\tau}^{t_0} \frac{c}{a(t)}\,dt = r \tag{19}$$

with $a(0) = 1$ by definition.

For a flat universe with zero cosmological constant, i.e., for $k = \Lambda = 0$ in the Einstein equations, $a(t)$ obeys $\dot{a} = A_1/\sqrt{a}$, and we get $a(t) = [1 - A_2(t_0 - t)]^{2/3}$ (see the Appendix, where the explicit value of the constants A_i is also given). With an elementary integration, and writing $t_0 - t = \tau$, we obtain that (19) is equivalent to

$$(1 - A_2\tau)^{1/3} = 1 - \frac{A_2 r}{3c} \tag{20}$$

and therefore

$$\tau = \frac{r}{c} - \frac{A_2 r^2}{3c^2} + \frac{A_2^2 r^3}{27c^3} \tag{21}$$

Inserting in this the value of A_2 (see the Appendix), we obtain

$$\tau = \frac{r}{c}\left[1 - \sqrt{\frac{MG}{2}}\,\frac{r}{c} + \left(\frac{MG}{6}\right)\frac{r^2}{c^2}\right] \tag{22}$$

The relation between recession speed and distance implied by the Calogero conjecture can be derived starting from (13), which, as remarked above, is an empirical, experimentally verified relation between the observed β (i.e., observed frequency of electromagnetic waves) and r (distance of the emitting object).

It should be mentioned that measuring the distance of far objects is not a simple task; for not too far ones, one can resort to measurement of parallaxes, but for very distant ones, a number of indirect techniques are used. In these cases, a varying value of h would imply all the distance estimates have to be modified (in a way I am not competent to discuss).

Remark 5. The distance of very far objects is typically estimated using luminosity of supernovae or other "macroscopic" measurements, but this does not by itself mean they will not depend on the value of h; e.g., for the total luminosity measurements it should be recalled that statistical mechanics (or thermodynamics) tells us the total amount of emitted energy—which is related to the total luminosity of a star—depends on the value of h as well: according to the Stefan–Boltzmann law, a body at temperature T emits a total energy $E(T)$ per unit of time given by $E(T) = (2\pi^4 k^4/15h^3c^2)T^4 := S(T)h^{-3}$. However, distances of objects which are not so far away can be directly measured, as mentioned above, via parallaxes; in particular, this can be done via satellite-based observations, thus reaching a much greater precision (and distance) than was previously possible. A parallax measurement is not influenced by a possibly different value of h, and this is another very good reason—together with the validity of the Hubble relation—to limit our discussion to not too high redshifts, i.e., to intermediate distances.

It will be convenient to introduce the dimensionless unit $\delta = r/\gamma \equiv Hr/c$. It should be stressed that $\delta << 1$: indeed the maximum possible value of δ is given by $\delta_{\max} = HR_0/c \simeq 8 \times 10^{-4}$.

With this notation, Eq. (13) yields

$$\beta^2 = \frac{1 - \delta}{1 + \delta} \tag{23}$$

and therefore (18) reads

$$w_{\mathrm{cal}} = \frac{(1 + \delta) - [a(t_0 - \tau)]^3(1 - \delta)}{(1 + \delta) + [a(t_0 - \tau)]^3(1 - \delta)} \tag{24}$$

In order to extract information from this we need a relation between $a(t_{\mathrm{em}})$

and δ; in other words, we need a model for the expansion of the universe and thus for the evolution of $R(t)$, or equivalently of $a(t)$.

4. HUBBLE–CALOGERO RELATION IN A FLAT UNIVERSE

Assuming a matter-dominated homogeneous and isotropic universe, the Einstein equations state that $R(t)$ obey

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} \tag{25}$$

where ρ is the density of matter, Λ is the cosmological constant, and k is a parameter which is equal to $(1, 0, -1)$ for a (closed, flat, open) universe. By writing $\rho = M/(4/3)\pi R^3$ with M the total mass-energy of the universe, we rewrite (25) as

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{2MG}{R^3} - \frac{k}{R^2} + \frac{\Lambda}{3} \tag{26}$$

Discussing $a(t) = R(t)/R_0$ rather than $R(t)$ would lead to essentially the same equations, with constants differing by an obvious factor.

Remark 6. The Einstein equations (25) are slightly different for a radiation-dominated universe, or for an intermediate case. Also, that of a homogeneous and isotropic universe is now known to be a not realistic hypothesis, observations pointing to a much more complex structure, but it is convenient to allow a simple discussion and illustrate the core of our argument.

We will assume a zero cosmological constant ($\Lambda = 0$), and discuss the flat case $k = 0$.

In this case the equation for $a(t)$ is solved in the Appendix, giving $a(t) = [1 - A_2(t_0 - t)]^{2/3}$ and hence $a(t_0 - \tau) = (1 - A_2\tau)^{2/3}$. Thus, (24) reads

$$w_{\text{cal}} = \frac{(1 + \delta) - (1 - A_2\tau)^2(1 + \delta)}{(1 + \delta) + (1 - A_2\tau)^2(1 + \delta)} \tag{27}$$

We can now use (20) and, writing

$$q = \frac{A_1}{2H} = \frac{A_2}{3H} = \sqrt{\frac{MG}{2R_0^3H^2}} = \sqrt{\frac{2\pi G\rho_0}{3H^2}} \tag{28}$$

we obtain finally the *Hubble–Calogero law* in the form

$$w_{\text{cal}} = \frac{(1 + \delta) - (1 - q\,\delta)^6(1 - \delta)}{(1 + \delta) + (1 - q\delta)^6(1 - \delta)} \tag{29}$$

If we assume a value of the order of (11), (11′) for H and the values $M =$

$4 \times 10^{54\pm8}$ g, $R_0 = 10^{28\pm2}$ cm, and $\rho = 10^{-30\pm2}$ which were used by Calogero to obtain his estimate on h, we obtain $q \simeq 1.5 \times 10^{2\pm1}$, with a very large error bar.

We will expand the Hubble–Calogero equation in a power series in δ around $\delta = 0$, as

$$w_{\text{cal}} = \sum_{k=0}^{\infty} \frac{f_k}{k!} \delta^k \tag{30}$$

The first few coefficients are given by

$$\begin{aligned} f_0 &= 0 \\ f_1 &= 1 + 3q \\ f_2 &= 3q^2 \\ f_3 &= -6q(3 + 9q + 8q^2) \\ f_4 &= -18q^2(2 + 12q + 17q^2) \end{aligned} \tag{31}$$

Let us focus our attention on the quadratic truncation of this, i.e.,

$$w_{\text{cal}} = (1 + 3q)\delta + (3/2)q^2\,\delta^2 \tag{32}$$

We notice the linear term presents a deviation from the standard Hubble law, which in this notation reads, simply $w = \delta$. This corresponds to a higher value of the Hubble constant (deduced from the immediate vicinity of our region) and thus to a faster present expansion rate $\dot{a}(t_0)/a(t_0)$. Moreover, the quadratic term has a positive coefficient. This corresponds to an expansion which is faster at greater distances, i.e., to an accelerating expansion.

Recalling the definition of δ, we can rewrite (32) in terms of r, v, and the standard Hubble constant H as

$$v_{\text{cal}} \approx (1 + 3q)Hr + \frac{3q^2H^2}{2c}\, r^2 \tag{33}$$

5. CONCLUSIONS AND FINAL REMARKS

We have seen that assuming the validity of the Calogero conjecture, and thus in particular the variation in time of the Planck constant according to (4), we are led (essentially, by the relativity formula for the Doppler shift) to reinterpret the standard existing data on the relation between the distance of celestial bodies and the redshift of the radiation they emitted: instead of the Hubble linear relation between distance and speed (thus an expansion that is spatially constant), they point to an expansion which is accelerating at higher distances.

The details of this derivation are questionable, and thus (29) represents only a very rough approximation: indeed, to derive it we assumed homogeneous expansion via a scale factor $a(t)$, and this is contradicted by our findings.

However, the qualitative result does not depend on how we proceeded to obtain formula (32): if we assume (4), then radiation has been emitted with a frequency higher than usually assumed, and thus the redshift is higher than in the standard interpretation. This effect increases more than linearly for greater distances.

Thus, the Calogero conjecture would imply that the existing observational data of the relation between redshift and distance, even for relatively small distances, should be reinterpreted as showing an accelerating expansion of the universe. This implies, in particular, that the age of the universe would be higher than estimated on the basis of the standard interpretation of cosmological data.

We stress that this modification of the Hubble law can be inferred in particular from data for relatively small distances, i.e., a region where direct measurement of distance is possible; this is therefore not subject to experimental uncertainities of the same nature as those necessarily entering the discussion of data referring to extremely far regions of the universe—for which distance measurements necessarily rely on a chain of physical theories—like those which were recently considered in various works [10–12] and also led to postulating accelerating expansion, but goes in the same direction.

This seems to call for further study on the interrelation between the possible cosmic origin of quantization and accelerated expansion, and in particular a possible nonzero value of the cosmological constant.

Remark 7. It should be noted that if the spatial density energy associated with Λ were not completely uniform but had small fluctuations, it could be at the origin of the "universal background noise"; however, the Calogero conjecture points to a possible origin of it due to fluctuations in the mass distribution even in the presence of a perfectly uniform structure of spacetime and distribution of the energy associated to the cosmological constant.

Remark 8. We also stress that a varying value of h would have some dramatic consequences for processes in early the universe, first of all on the processes responsible for the cosmic background radiation and for nucleosynthesis. Thus, one could think of testing the Calogero conjecture (4) against these processes, i.e., study the consequences of (4) for the present temperature of the background radiation and for the abundance of heavy elements. However, such an extrapolation of (4) to very early times in the universe would not be justified: the physical idea at the basis of (4) is that quantum behavior is due to a gravitational effect, essentially to fluctuations in the gravitational force due to interaction with distant masses. In a different phase of the universe, i.e., immediately after the big bang, the same mechanism could give rise to a law quite different from (4). Thus, we cannot extrapolate (4) to such times unless this is justified by a detailed study of the fluctuations in question; needless to say, this is a very hard task, both in the frame of the state of the present universe and even more in that of the universe immediately after the big bang.

APPENDIX

In this Appendix we derive the solution of the Einstein equations (25) for a flat Universe ($k = 0$) and a vanishing cosmological constant ($\Lambda = 0$). We consider the equations for the scale factor $a(t)$; obviously the radius $R(t)$ is readily recovered by recalling that $R(t) = a(t)R_0$. We denote the present time by t_0, so that $R(t_0) = R_0$, and also write $t = t_0 - \tau$. We also notice that $\rho(t) = \rho_0/[a(t)]^3$, where of course $\rho_0 = \rho(t_0)$.

Thus the equations are simply

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_0}{3a^3} \equiv \frac{A_1^2}{a^3} \tag{A.1}$$

We rewrite this as $\dot{a} = A_1/\sqrt{a}$, namely $\sqrt{a}\, da = A_1\, dt$, and solve it by separation of variables. This yields

$$a(t) = (A_2 t + b_0)^{2/3} \tag{A.2}$$

where b_0 is an arbitrary constant and $A_2 = (3/2)A_1$. We impose now the boundary condition $a(t_0) = 1$, which means $b_0\ A_2 t_0$: thus (A.3) is rewritten as

$$a(t) = [1 - A_2(t_0 - t)]^{2/3} \tag{A.3}$$

and we have in particular that

$$a(t_0 - \tau) = (1 - A_2\tau)^{2/3} \tag{A.4}$$

Here $A_2 = \sqrt{6\pi G\rho_0} = \sqrt{9MG/2R_0^3}$ and $M = (4/3)\pi R_0^3\rho_0$ is the total mass of the universe.

ACKNOWLEDGMENTS

I would like to thank Orlando Ragnisco for reviving my interest in the Calogero conjecture. I also thank Alberto Ibort for his invitation in Madrid and all the Departamento de Matematicas of the Universidad Carlos III for their hospitality.

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